Instructions: Try these problems from the textbook as practice (answers below).

Problems

1. Suppose you start at the origin, move along the x-axis a distance of 4 units in the positive direction, and then move downward a distance of 3 units. What are the coordinates of your position?

3. Which of the points $A(-4, 0, -1)$, $B(3, 1, -5)$, and $C(2, 4, 6)$ is closest to the yz-plane? Which point lies in the xz-plane?

5. What does the equation $x = 4$ represent in $\mathbb{R}^2$? What does it represent in $\mathbb{R}^3$? Illustrate with sketches.

7. Describe and sketch the surface in $\mathbb{R}^3$ is represented by the equation $x + y = 2$.

9. Find the lengths of the sides of the triangle PQR where $P(3, -2, -3)$, $Q(7, 0, 1)$ and $R(1, 2, 1)$. Is it a right triangle? Is it an isosceles triangle?

11. Determine whether the points lie on a straight line.

    (a) $A(2, 4, 2)$, $B(3, 7, -2)$, $C(1, 3, 3)$.

    (b) $D(0, -5, 5)$, $E(1, -2, 4)$, $F(3, 4, 2)$

13. Find an equation of the sphere with centre $(-3, 2, 5)$ and radius 4. What is the intersection of this sphere with the yz-plane?

15. Find an equation of the sphere that passes through the point $(4, 3, -1)$ and has centre $(3, 8, 1)$.

17-20. Show that the equation represents a sphere, and find its centre and radius.

17. $x^2 + y^2 + z^2 - 2x - 4y + 8z = 15$

19. $2x^2 + 2y^2 + 2z^2 = 8x - 24z + 1$

25-38. 5BDescribe in words the region of $\mathbb{R}^3$ represented by the equation(s) or inequality.

25. $x = 5$

27. $y < 8$

29. $0 \leq z \leq 6$

31. $x^2 + y^2 = 4$, $z = -1$

33. $x^2 + y^2 + z^2 = 4$

35. $1 \leq x^2 + y^2 + z^2 \leq 5$

37. $x^2 + z^2 \leq 9$

39-42. Write inequalities to describe the region.

39. The region between the yz-plane and the vertical plane $x = 5$.

41. The region consisting of all points between (but not on) the spheres of radius r and R centred at the origin, where $r < R$.

45. Find an equation of the set of all points equidistant from the points $A(-1, 5, 3)$ and $B(6, 2, -2)$. Describe the set.

Answers

1. $(4, 0, -3)$
3. C; A
5. A line parallel to the y-axis and 4 units to the right of it; a vertical plane parallel to the yz-plane and 4 units in front of it.
7. A vertical plane that intersects the xy-plane in the line $y = 2 - x$, $z = 0$.
9. $d(P, Q) = 6$, $d(Q, R) = 2\sqrt{10}$, $d(R, P) = 6$; isosceles triangle
11. No; Yes
13. $(x + 3)^2 + (y - 2)^2 + (z - 5)^2 = 16$; $(y - 2)^2 + (z - 5)^2 = 7$, $x = 0$ (a circle)
15. $(x - 3)^2 + (y - 8)^2 + (z - 1)^2 = 30$
17. $(1, 2, -4)$, 6
19. $(2, 0, -6)$, $9/\sqrt{2}$
25. A plane parallel to the yz-plane and 5 units in front of it
27. A half-space consisting of all points to the left of the plane $y = 8$
29. All points on or between the horizontal planes $z = 0$ and $z = 6$
31. All points on a circle with radius 2 with centre on the z-axis that is contained in the plane $z = -1$
33. All points on a sphere with radius 2 and centre $(0, 0, 0)$.
35. All points on or between spheres with radii 1 and $\sqrt{5}$ and centred at $(0, 0, 0)$.
37. All points on or inside a circular cylinder or radius 3 with axis the y-axis.
39. $0 < x < 5$
41. $r^2 < x^2 + y^2 + z^2 < R^2$
45. $14x - 6y - 10z = 9$, a plane perpendicular to AB (the straight line segment joining points A and B)